Q2/ In a group $(G,+) \ni G=\{a+\sqrt[3]{2}\,b \mid a,b\in \mathbb{Z}\}$, find identity and inverse elements.

solution: ① Identity element

let $e = a_1+\sqrt[3]{2}\,b_1$, so $\forall a+\sqrt[3]{2}\,b \in G$

$$a*e = e*a = a$$

$$(a+\sqrt[3]{2}\,b)+(a_1+\sqrt[3]{2}\,b_1) = (a+\sqrt[3]{2}\,b)$$

$$(a+a_1)+\sqrt[3]{2}\,(b+b_1) = a+\sqrt[3]{2}\,b$$

$$\therefore\ a+a_1 = a \Rightarrow a_1 = 0$$

$$b+b_1 = b \Rightarrow b_1 = 0$$

$$\therefore\ e = a_1+\sqrt[3]{2}\,b_1 = 0+\sqrt[3]{2}\cdot 0$$

② Inverse element

let $a+\sqrt[3]{2}\,b\in G$ and inverse element is $a_1+\sqrt[3]{2}\,b_1$

$$a*a^{-1} = a^{-1}*a = e$$

$$(a+\sqrt[3]{2}\,b)+(a_1+\sqrt[3]{2}\,b_1) = 0+\sqrt[3]{2}\cdot 0$$

$$(a+a_1)+\sqrt[3]{2}\,(b+b_1) = 0+\sqrt[3]{2}\cdot 0$$

$$a+a_1 = 0 \Rightarrow a_1 = -a$$

$$b+b_1 = 0 \Rightarrow b_1 = -b$$

$$\therefore\ a^{-1} = a_1+\sqrt[3]{2}\,b_1 = -a-\sqrt[3]{2}\,b = -(a+\sqrt[3]{2}\,b)$$