Properties of absolute values

Theorem: Let x, y $\in$ R, then

1- $|-x| = x$

2- $|x-y| = |y-x|$

3- $|x|^2 = x^2$

4- $|x| = \sqrt{x^2}$.

5- $|x.y| = |x||y|$.

6- $\left|x/y\right| = |x|/|y|$ if $y \neq 0$.

7- $|x| \le b \Leftrightarrow -b \le x \le b$ where b > 0 .

8- $|x+y| \le |x| + |y|$.

9- $|x| - |y| \le |x-y|$.

10- $||x| - |y|| \le |x-y|$.

11- $|x^n| = |x|^n$ where n nonnegative integer.

Proof. 2) From property 1 applied to x−y we have 2.

3) $\because$ $|x|^2 = |x|.|x|$ then,

i) If $x \ge 0$ then $|x| =$ x , $\therefore |x|^2 = x.x = x^2$

ii) if $x < 0$ then $|x| =$ - x , $\therefore |x|^2 = (-x).(-x) = x^2$

4) from 3, باخذ الجذر التربيعي للطرفين

8) from (7), $|x| \le |x| \Leftrightarrow -|x| \le x \le |x|$ …..(i)

Also, $|y| \le |y| \Leftrightarrow -|y| \le y \le |y|$..... (ii)

$\therefore$ (i) & (ii) بجمع

$$-(|x| + |y|) \le x + y \le (|x| + |y|)$$

$$\therefore \quad |x+y| \le (|x| + |y|)$$

11) if n=0 فان العبارة متحققة

and if n>0 then,

$$|x^n| = |x.x.x...x| = |x|.|x|.|x| \ldots |x| = |x|^n$$