**Q1/ (Q,+ , .) is a ring.**

1. a +b $\in $ Q $∀$ a , b $\in $ Q. ( closure law of addition)
2. (a+b)+c = a + (b+c) $∀$ a,b,c $\in $ Q. ( Associative law of addition)
3. $∃$ 0 $\in $ Q such that a+0 = a. (existence of additive identity)
4. $∀$a $\in $ Q, $∃$x $\in $ Q such that a+ x =0. (existence of additive inverse)
5. a + b = b +a $∀$ a , b , $\in $ Q. ( commutative law of addition)
6. a .b $\in $ Q$ ∀$ a , b , $\in $ Q ( closure law of multiplication)
7. a (b+c)=a.b + a.c, and (a+b) . c = a . c + b.c $∀$ a,b,c $\in $ Q. ( Associative law of multiplication)
8. (a . b ) . c = a . ( b . c ) $∀$ a,b,c $\in $ Q. ( distributive laws)

(Q, + , .) is a ring.

Commutative: a.b=b.a $∀$ a,b $\in $ Q.

Identity: a.1=1.a=a $∀$ a $\in $ Q.

Invertible: $∀$a $\in $ Q, $∃b=a^{-1}$ $\in $ Q such that a,b=1 every element in Q has inverse except 0.

**Q2/ (3Z,+ , .) is a ring.**

By the same way from 1 - 8 in Q1.

Commutative: 3(a .b)=3(b .a) $∀$ a,b $\in $ 3Z.

 3ab = 3ba

Identity: Let 3Z = {………-9 , -6, -3, 0 ,3 , 6 , 9 ……... } without identity.

(3Z,+ , .) ring with no identity and no invertible.

**Q3/ (**$\left[\begin{matrix}a&b\\c&d\end{matrix}\right]$, +, .) **is a ring.**

By the same way from 1 - 8 in Q1.

Commutative law : Let we have x = $\left[\begin{matrix}a&b\\c&d\end{matrix}\right]$ and y = $\left[\begin{matrix}e&f\\g&h\end{matrix}\right]$ , then

$\left[\begin{matrix}a&b\\c&d\end{matrix}\right]\left[\begin{matrix}e&f\\g&h\end{matrix}\right]\ne $ $\left[\begin{matrix}e&f\\g&h\end{matrix}\right]\left[\begin{matrix}a&b\\c&d\end{matrix}\right]$

$\left[\begin{matrix}ae+bg&af+bh\\ce+dg&cf+dh\end{matrix}\right]\ne $ $\left[\begin{matrix}ea+fc&eb+fd\\ga+hc&gd+hd\end{matrix}\right]$ , not commutative.

Identity: $\left[\begin{matrix}a&b\\c&d\end{matrix}\right]\left[\begin{matrix}1&0\\0&1\end{matrix}\right]=$ $\left[\begin{matrix}1&0\\0&1\end{matrix}\right]\left[\begin{matrix}a&b\\c&d\end{matrix}\right]$

 $\left[\begin{matrix}a&b\\c&d\end{matrix}\right]=$ $\left[\begin{matrix}a&b\\c&d\end{matrix}\right]$ with identity $\left[\begin{matrix}1&0\\0&1\end{matrix}\right]$.

**Q4/** Let R be a ring, then

1. a.0=0.a=0
2. (-a) .b = a. (-b) = - (a.b)
3. (-a) (-b) = a.b.
4. a. (b-c)= a.b – a.c for all a,b, c $\in $ R.

**proof: -**

1. a.0=a.(0+0)=a.0+a.0=0
2. 0 = 0.b=(a + (-a))b= ab + (-a)b implies that (-a)b = -(ab)
3. (-a) (-b) = - (a. (-b)) = - ( - ( a.b)) = a.b
4. a.(b-c) = a. [ b + (-c)] = a.b + a. (-c) = a.b – a.c

 **Q5/** Let (Z,+,.), (Q,+,.), (R,+,.) be a rings. Show that are integraldomains.

 Proof: - $∀$ a,b $\in $ Z , Q, R. If a.b = 0 , then either a =0 or b =0.

 **Q6/ Let** of ( Z , + , .) be a ring. prove ( 2Z, + , .) is a subring .

**Proof: -** 2Z = {………-6 , -4, -2, 0 ,2 , 4 , 6 ……... }

 Z = {………-3 , -2, -1, 0 ,1 , 2 , 3 ……... }.

1. $∀$ a,b $\in $ 2Z implies that a – b $\in $ 2Z $⊂$Z.
2. $∀$ a,b $\in $ 2Z implies that a . b $\in $ 2Z $⊂$Z.

( 2Z, + , .) is a subring.

**Q7/** Let Z = {………-3 , -2, -1, 0 ,1 , 2 , 3 ……... }.

 2Z={………-6 , -4, -2, 0 ,2 , 4 , 6 , 8……... },

 prove ( 2Z, + , .) is subring and ideal.

1. $∀$ a,b $\in $ 2Z implies that a – b $\in $ 2Z $⊂$Z.
2. $∀$ a,b $\in $ 2Z implies that a . b $\in $ 2Z $⊂$Z.

 ( 2Z, + , .) is a subring.

1) $∀$ a,b $\in $ 2Z implies that a . b $\in $ 2Z $⊂$Z.

1. $∀$ a $\in $ 2Z, r$\in $ Z implies that a. r $\in $ 2Z $⊂$Z.

Example: a =2 , b =4 , r =3 implies that a . b = 2 . 4 = 8 $\in $ 2Z

a.r = 2.3 = 6 $\in $ 2Z

2Z is ideal.