**Q1/ (Q,+ , .) is a ring.**

1. a +b Q a , b Q. ( closure law of addition)
2. (a+b)+c = a + (b+c) a,b,c Q. ( Associative law of addition)
3. 0 Q such that a+0 = a. (existence of additive identity)
4. a Q, x Q such that a+ x =0. (existence of additive inverse)
5. a + b = b +a a , b , Q. ( commutative law of addition)
6. a .b Q a , b , Q ( closure law of multiplication)
7. a (b+c)=a.b + a.c, and (a+b) . c = a . c + b.c a,b,c Q. ( Associative law of multiplication)
8. (a . b ) . c = a . ( b . c ) a,b,c Q. ( distributive laws)

(Q, + , .) is a ring.

Commutative: a.b=b.a a,b Q.

Identity: a.1=1.a=a a Q.

Invertible: a Q, Q such that a,b=1 every element in Q has inverse except 0.

**Q2/ (3Z,+ , .) is a ring.**

By the same way from 1 - 8 in Q1.

Commutative: 3(a .b)=3(b .a) a,b 3Z.

3ab = 3ba

Identity: Let 3Z = {………-9 , -6, -3, 0 ,3 , 6 , 9 ……... } without identity.

(3Z,+ , .) ring with no identity and no invertible.

**Q3/ (**, +, .) **is a ring.**

By the same way from 1 - 8 in Q1.

Commutative law : Let we have x = and y = , then

, not commutative.

Identity:

with identity .

**Q4/** Let R be a ring, then

1. a.0=0.a=0
2. (-a) .b = a. (-b) = - (a.b)
3. (-a) (-b) = a.b.
4. a. (b-c)= a.b – a.c for all a,b, c R.

**proof: -**

1. a.0=a.(0+0)=a.0+a.0=0
2. 0 = 0.b=(a + (-a))b= ab + (-a)b implies that (-a)b = -(ab)
3. (-a) (-b) = - (a. (-b)) = - ( - ( a.b)) = a.b
4. a.(b-c) = a. [ b + (-c)] = a.b + a. (-c) = a.b – a.c

**Q5/** Let (Z,+,.), (Q,+,.), (R,+,.) be a rings. Show that are integraldomains.

Proof: - a,b Z , Q, R. If a.b = 0 , then either a =0 or b =0.

**Q6/ Let** of ( Z , + , .) be a ring. prove ( 2Z, + , .) is a subring .

**Proof: -** 2Z = {………-6 , -4, -2, 0 ,2 , 4 , 6 ……... }

Z = {………-3 , -2, -1, 0 ,1 , 2 , 3 ……... }.

1. a,b 2Z implies that a – b 2Z Z.
2. a,b 2Z implies that a . b 2Z Z.

( 2Z, + , .) is a subring.

**Q7/** Let Z = {………-3 , -2, -1, 0 ,1 , 2 , 3 ……... }.

2Z={………-6 , -4, -2, 0 ,2 , 4 , 6 , 8……... },

prove ( 2Z, + , .) is subring and ideal.

1. a,b 2Z implies that a – b 2Z Z.
2. a,b 2Z implies that a . b 2Z Z.

( 2Z, + , .) is a subring.

1) a,b 2Z implies that a . b 2Z Z.

1. a 2Z, r Z implies that a. r 2Z Z.

Example: a =2 , b =4 , r =3 implies that a . b = 2 . 4 = 8 2Z

a.r = 2.3 = 6 2Z

2Z is ideal.