Trapezoidal rule

In [mathematics](https://en.wikipedia.org/wiki/Mathematics), and more specifically in [numerical analysis](https://en.wikipedia.org/wiki/Numerical_analysis), the **trapezoidal rule**(also known as the **trapezoid rule** or **trapezium rule**) is a technique for approximating the [definite integral](https://en.wikipedia.org/wiki/Integral)

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The function *f*(*x*) (in blue) is approximated by a linear function (in red).





# Example 1.

Approximate the integral of f(*x*) = *x*3 on the interval [1, 2] with four subintervals.

First, *h* = (2 - 1)/4 = 0.25, and thus we calculate:

½⋅(f(1) + 2⋅(f(1.25) + f(1.5) + f(1.75)) + f(2))⋅0.25
= ½⋅(13 + 2⋅(1.253 + 1.53 + 1.753) + 23)⋅0.25 = 3.796875

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## If n=8

we set *h* = (2 - 1)/8 = 0.125, and thus we calculate:

½⋅(f(1)+2⋅(f(1.125)+f(1.25)+f(1.375)+f(1.5)+f(1.625)+f(1.75)+f(1.875))+f(2))⋅0.125
= ½⋅(13+2⋅(1.1253+1.253+1.3753+1.53+1.6253+1.753+1.8753)+23)⋅0.25 = 3.76171875

The second approximation is much closer to the correct answer of 3.75.



## ÙØªÙØ¬Ø© Ø¨Ø­Ø« Ø§ÙØµÙØ± Ø¹Ù âªtrapezoidal method exampleâ¬â

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function It=trapezios(f,a,b,N)

h=(b-a)/N;

It=0;

for k=1:(N-1)

 x=a+h\*k;

 It=It+feval(f,x);

 end

It=h\*(f(a)+f(b))/2+h\*It;

End

## Example

Evaluate the integral x^4 within limits -3 to 3 using Trapezoidal rule.

## Solution

Let y(x)=x^4

here a=-3 and b=3

therefore (b-a)=6

let ‘n’ be the number of intervals. assume n=6 in this case.

also h=(b-a)/n = 6/6 =1

x: -3  -2  -1  0  1  2  3

y: 81  16  1  0  1  16  81

According to trapezoidal rule:

answer= (h/2)\*[(y1+y7)+2\*(y2+y3+y4+y5+y6)]

answer=(1/2)\*[(81+81)+2\*(16+1+0+1+16)]

answer=115.



## References

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