

probab. function
 الدالة الاحتمالية

discrete r.v.

prob. mass. func.
 دالة الكتلة الاحتمالية
 p.m.f

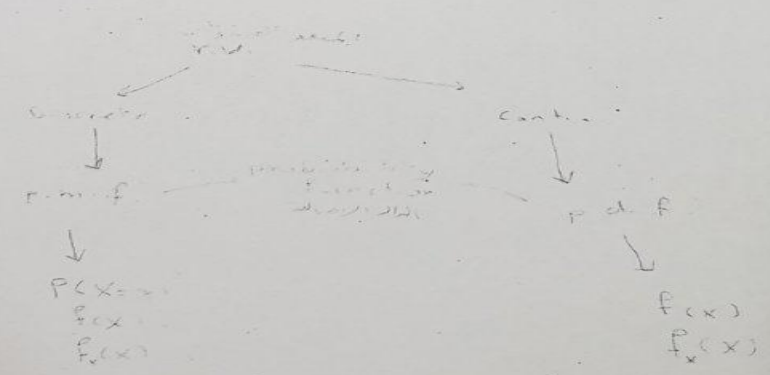
المركب / p.m.f
 $P(x)$
 $P(X=x)$
 $f(x)$
 $f_x(x)$

C.d.f
 $F(x)$

continuous r.v.

prob. density func.
 p.d.f
 دالة الكثافة الاحتمالية

المركب / p.d.f
 $f(x)$
 $f_x(x)$



Cumulative density func.
 C.d.f.
 $F(x)$

Def. 2 : Discrete Random Variable.

A random variable X that takes on distinct values only is called a discrete random variable.

Ex.

Suppose that our experiment is tossing a fair coin. Let X denote the number of heads in the experiment. The sample space in this case is $S = \{\text{head (H)}, \text{tail (T)}\}$.

then the variable takes on either 0 or 1, thus X is a discrete random variable with,

$$\{T\} \Rightarrow X = 0$$

$$\{H\} \Rightarrow X = 1$$

Def. Probability Mass Function (p.m.f):

Let X be a discrete random variable and let $x_1, x_2, \dots$ be distinct values that X may assume, Then the function, $p(x)$ defined by:

$$p(x) = \begin{cases} p(X=x_i) & , X=x_i \quad , i=1, 2, \dots \\ 0 & , X \neq x_i \end{cases}$$

is defined to be the probability mass function of the random variable X , $p(x_i)$ is called the probability mass at value x_i ,

probability of
properties of the p.m.f.

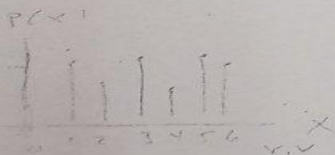
$$1- 0 \leq p(x_i) \leq 1 \quad , i=1, 2, \dots$$

$$2- \sum_{i=1}^{\infty} p(x_i) = 1$$



كيفية رسم الدالة $p(x)$
هي عبارة عن نقاط منفصلة
على محور x فوق المحور الأفقي x
الارتفاع في المحور y يساوي $p(x_i)$

كيفية رسم الدالة $p(x)$ بشكل نقاط منفصلة وذات ارتفاعات مختلفة
عندما تكون كثيرة تقريبا، كما تكون خطية عند



ex (4) For example, find p.m.f. and draw it.
 A coin is thrown once, X represents the r.v. and represents the number of heads.

$$X = 0, 1$$

$$S = \{H, T\} \quad n(s) = 2$$

$$X = 0 \Rightarrow \{T\} \Rightarrow P(X=0) = \frac{1}{2}$$

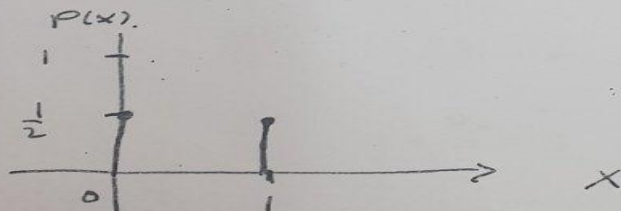
$$X = 1 \Rightarrow \{H\} \Rightarrow P(X=1) = \frac{1}{2}$$

$$P(X) = \begin{cases} \frac{1}{2} & X=0 \\ \frac{1}{2} & X=1 \\ 0 & \text{o.w.} \end{cases}$$

$$\sum_{x=0}^1 P(X) = P(0) + P(1) = \frac{1}{2} + \frac{1}{2} = 1$$

$$0 \leq P(0), P(1) \leq 1$$

$\therefore P(X)$ is p.m.f.



Ex. ⑧ Two coins are thrown once, the r.v. X represents the number of heads. find the p.m.f.

$$S = \{HH, HT, TH, TT\}. \quad n(S) = 4.$$

$$X = 0, 1, 2.$$

$$P(X) = \begin{cases} \frac{1}{4} \\ \frac{2}{4} \\ \frac{1}{4} \\ 0 \end{cases}$$

$$X = 0 \rightarrow \{T\}. \quad P(X=0) = \frac{1}{4}$$

$$X = 1 \rightarrow \{TH, HT\}, \quad P(X=1) = \frac{2}{4}$$

$$X = 2 \rightarrow \{HH\}, \quad P(X=2) = \frac{1}{4}$$

o.w

$$\sum_{x=0}^2 P(X) = P(0) + P(1) + P(2) = \frac{1}{4} + \frac{2}{4} + \frac{1}{4} = \boxed{1}$$

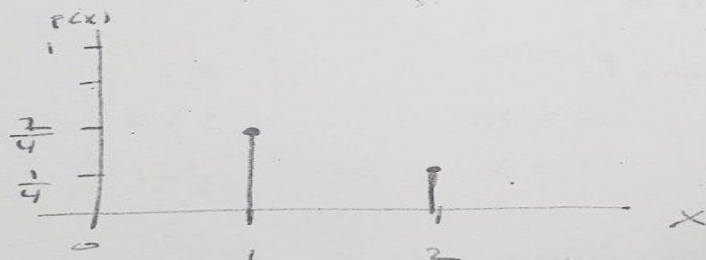
$$= P(X) \text{ is } 1$$

$$\Rightarrow P(0), P(1), P(2) \leq 1$$

o.w

$\therefore P(X)$ is the p.m.f.

the figure



Sample Space	X	P(X)
TT	0	$P(X=0) = 1/4$
TH	1	$P(X=1) = 2/4$
HT	1	
HH	2	$P(X=2) = 1/4$

$n=4$, r.v. $X = 0, 1, 2$

⑨ for example (5) find the p.m.f.

ex. 9

Three coins are thrown once. Let X be the number of heads obtained. then the values taken by X are 0, 1, 2, 3. To find the p.m.f of X ?

$$s = 2^3 = 8$$

$$S = \{HHH, HHT, HTH, THH, TTH, THT, HTT, TTT\}$$

$$X=0 \Rightarrow \{TTT\} \Rightarrow P(X=0) = \frac{1}{8}$$

$$X=1 \Rightarrow \{TTH, THT, HTT\} \Rightarrow P(X=1) = \frac{3}{8}$$

$$X=2 \Rightarrow \{HHT, HTH, THH\} \Rightarrow P(X=2) = \frac{3}{8}$$

$$X=3 \Rightarrow \{HHH\} \Rightarrow P(X=3) = \frac{1}{8}$$

$$p(x) = \begin{cases} \frac{1}{8} & , X=0 \\ \frac{3}{8} & , X=1 \\ \frac{3}{8} & , X=2 \\ \frac{1}{8} & , X=3 \\ 0 & , \text{otherwise} \end{cases}$$

no	sample	X	p(x)
1	TTT	0	$p(X=0) = \frac{1}{8}$
2	HTT THT TTH	1	$p(X=1) = \frac{3}{8}$
3	THH HTH HHT	2	$p(X=2) = \frac{3}{8}$
4	HHH	3	$p(X=3) = \frac{1}{8}$
n=8			$\sum p(x) = 1$

Notes: (1) $0 \leq P(X_i) \leq 1$

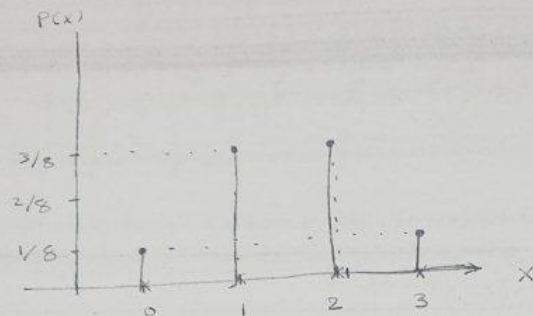
$$\Rightarrow 0 \leq P(X_1), P(X_2), P(X_3), P(X_4) \leq 1$$

$$(2) \sum_{\text{all } X} P(X_i) = 1$$

$$\begin{aligned} \sum_{X=0}^3 P(X_i) &= P(X=0) + P(X=1) + P(X=2) + P(X=3) \\ &= \frac{1}{8} + \frac{3}{8} + \frac{3}{8} + \frac{1}{8} = \boxed{1} \end{aligned}$$

Hence $p(x)$ is p.m.f.

Figure of this function.



ex. p. (10) ^{ex. 10}
ex. 11

Suppose that we throw one die, Let X be the number shown by this die, Find the probability Mass function of this experiment.

$X = 1, 2, 3, 4, 5, 6$.

$$P(X=1) = 1/6$$

$X=1$

$$P(X=2) = 1/6$$

$X=2$

$$P(X=3) = 1/6$$

$X=3$

$\vdots$

$\vdots$

$$P(X=6) = 1/6$$

$X=6$

in general.

$$P(X=x) = \begin{cases} \frac{1}{6} & , \quad x = 1, 2, 3, 4, 5, 6 \\ 0 & \text{o.w.} \end{cases}$$

$$\sum_{x=1}^6 P(X) = P(1) + P(2) + P(3) + P(4) + P(5) + P(6)$$

$$= \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6}$$

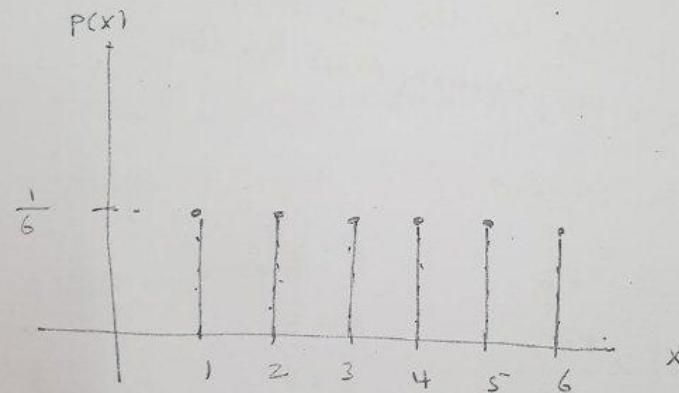
$$= \frac{6}{6}$$

$$\sum_{x=1}^6 P(x) = 1$$

$$0 \leq P(x_i) \leq 1$$

$$0 \leq P(X_1), P(X_2), \dots, P(X_6) \leq 1$$

$\therefore P(X)$ is a p. m. f.



Ex (3) Two dice are thrown once. Define a.r.v. X to be the sum of the two numbers shown by the two dice. To find the p.m.f of X we get the sample space as follows:

$$S = \left\{ \begin{array}{cccccc} (1,1), (1,2), (1,3), (1,4), (1,5), (1,6) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ (6,1), \dots, \dots, \dots, \dots, (6,6) \end{array} \right\}, n(S) = 36$$

$$X = 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12$$

$$\{(1,1)\} \Rightarrow X = 1+1 = 2 \quad P(X=2) = \frac{1}{36}$$

$$\{(2,1), (1,2)\} \Rightarrow X = 2+1 = 1+2 = 3 \quad P(X=3) = \frac{2}{36}$$

$$\{(1,3), (3,1), (2,2)\} \Rightarrow X = 1+3 = 3+1 = 2+2 = 4 \quad P(X=4) = \frac{3}{36}$$

$$\{(1,4), (4,1), (2,3), (3,2)\} \Rightarrow X = \begin{array}{l} 1+4 \\ 4+1 \\ 2+3 \\ 3+2 \end{array} = 5 \quad P(X=5) = \frac{4}{36}$$

$$\{(1,5), (5,1), (2,4), (4,2), (3,3)\} \Rightarrow X = \begin{array}{l} 1+5 \\ 5+1 \\ 2+4 \\ 4+2 \\ 3+3 \end{array} = 6 \quad P(X=6) = \frac{5}{36}$$

$$\{(1,6), (6,1), (2,5), (5,2), (3,4), (4,3)\} \Rightarrow X = \begin{array}{l} 1+6 \\ 6+1 \\ 2+5 \\ 5+2 \\ 3+4 \\ 4+3 \end{array} = 7 \quad P(X=7) = \frac{6}{36}$$

$$\{(2,6), (6,2), (3,5), (5,3), (4,4)\} \Rightarrow X = \begin{array}{l} 2+6 \\ 6+2 \\ 3+5 \\ 5+3 \\ 4+4 \end{array} = 8 \quad P(X=8) = \frac{5}{36}$$

$$\{(3,6), (6,3), (4,5), (5,4)\} \Rightarrow X = \begin{array}{l} 3+6 \\ 6+3 \\ 4+5 \\ 5+4 \end{array} = 9 \quad P(X=9) = \frac{4}{36}$$

$$\{(4,6), (6,4), (5,5)\} \Rightarrow X = \begin{array}{l} 4+6 \\ 6+4 \\ 5+5 \end{array} = 10 \quad P(X=10) = \frac{3}{36}$$

$$\{(5,6), (6,5)\} \Rightarrow X = \begin{array}{l} 5+6 \\ 6+5 \end{array} = 11 \quad P(X=11) = \frac{2}{36}$$

$$\{(6,6)\} \Rightarrow X = 6+6 = 12 \quad P(X=12) = \frac{1}{36}$$

(5)

In general.

$$X = 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12.$$

$$P(X=2) = 1/36, X=2$$

$$P(X=3) = 2/36, X=3$$

$$P(X=4) = 3/36, X=4$$

$$P(X=5) = 4/36$$

$$P(X=6) = 5/36$$

$$P(X=7) = 6/36$$

$$P(X=8) = 5/36$$

$$P(X=9) = 4/36$$

$$P(X=10) = 3/36$$

$$P(X=11) = 2/36$$

$$P(X=12) = 1/36, X=12$$

In general.

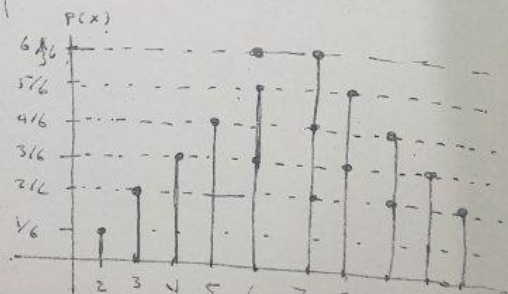
$$P(X) = \begin{cases} \frac{X-1}{36} & , X=2,3,4,5,6,7 \\ \frac{13-X}{36} & , X=8,9,10,11,12 \\ 0 & , 0, \infty \end{cases}$$

$$\sum_{x=2}^{12} P(X) = \frac{1}{36} + \frac{2}{36} + \dots + \frac{1}{36} = 1$$

$$0 \leq P(X_1), P(X_2), \dots, P(X_{12}) \leq 1$$

$\therefore P(X)$ is the p.m.f.

the figure



Ex. 1.62

Two dice are thrown, the r.v. X represent the difference between the two numbers, find the p.m.f.

$s.s. = 36$

- $X=0 \rightarrow \{(5,5), (6,6), (4,4), (3,3), (2,2), (1,1)\} \rightarrow P(X=0) = \frac{6}{36}$
- $X=1 \rightarrow \{(6,5), (5,4), (4,3), (3,2), (2,1)\} \rightarrow P(X=1) = \frac{5}{36}$
- $X=2 \rightarrow \{(6,4), (5,3), (4,2), (3,1)\} \rightarrow P(X=2) = \frac{4}{36}$
- $X=3 \rightarrow \{(6,3), (5,2), (4,1)\} \rightarrow P(X=3) = \frac{3}{36}$
- $X=4 \rightarrow \{(6,2), (5,1)\} \rightarrow P(X=4) = \frac{2}{36}$
- $X=5 \rightarrow \{(6,1)\} \rightarrow P(X=5) = \frac{1}{36}$
- $X=-1 \rightarrow \{(5,6), (4,5), (3,4), (2,3), (1,2)\} \rightarrow P(X=-1) = \frac{5}{36}$
- $X=-2 \rightarrow \{(4,6), (3,5), (2,4), (1,3)\} \rightarrow P(X=-2) = \frac{4}{36}$
- $X=-3 \rightarrow \{(3,6), (2,5), (1,4)\} \rightarrow P(X=-3) = \frac{3}{36}$
- $X=-4 \rightarrow \{(2,6), (1,5)\} \rightarrow P(X=-4) = \frac{2}{36}$
- $X=-5 \rightarrow \{(1,6)\} \rightarrow P(X=-5) = \frac{1}{36}$

the p.m.f is:

$P(X) =$	$\frac{6}{36}$	$X=0$
	$\frac{5}{36}$	$X=1, -1$
	$\frac{4}{36}$	$X=2, -2$
	$\frac{3}{36}$	$X=3, -3$
	$\frac{2}{36}$	$X=4, -4$
	$\frac{1}{36}$	$X=5, -5$
	0	$X=6$

$\begin{matrix} & 1 & 2 & 3 & 4 & 5 & 6 \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{pmatrix} 0 & -1 & 2 & -3 & -4 & -5 \\ 1 & 0 & -1 & -2 & -3 & -4 \\ 2 & 1 & 0 & -1 & -2 & -3 \\ 3 & 2 & 1 & 0 & -1 & -2 \\ 4 & 3 & 2 & 1 & 0 & -1 \\ 5 & 4 & 3 & 2 & 1 & 0 \end{pmatrix} \end{matrix}$

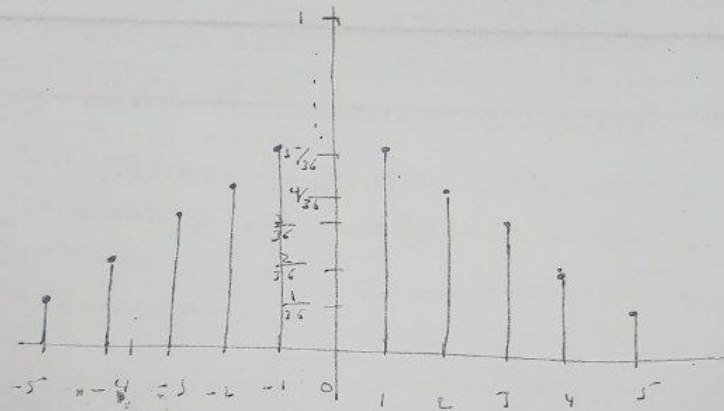
$X = -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5$

$$\begin{aligned}
 \textcircled{1} \quad \sum_{x=-5}^5 p(x) &= p(-5) + p(-4) + \dots + p(5) \\
 &= \frac{5}{16} + \frac{4}{16} + \frac{3}{16} + \frac{2}{16} + \frac{1}{16} + \frac{1}{16} + \frac{2}{16} + \frac{3}{16} + \frac{4}{16} + \frac{5}{16} \\
 &= 1
 \end{aligned}$$

$$\textcircled{2} \quad 0 \leq p(-5), p(-4), \dots, p(4), p(5) \leq 1$$

$\therefore p(x)$ is p.m.f.

the figure



Def. Cumulative Distribution Function (C.d.f)
or Distribution function.

The cumulative distribution function of a discrete random variable is denoted by $F(x)$ and is defined by.

$$F(x) = P(X \leq x) = \sum_{x_i \leq x} p(x_i)$$

properties of c.d.f.

(1) $\lim_{x \rightarrow \infty} F(x) = 1$

(2) $\lim_{x \rightarrow -\infty} F(x) = 0$ دالة التوزيع التراكمي هي دالة غير متناقصة اي دالة متزايدة

(3) $F(x_1) \leq F(x_2)$ if $x_1 < x_2$
 $F(x_i) \leq F(x_{i+1})$

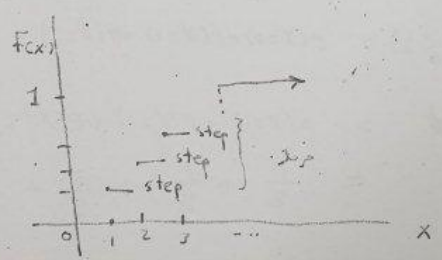
(4) $P(a < X \leq b) = F(b) - F(a)$ if $a \leq b$
 $P(x_i) = F(x_i) - F(x_{i-1})$

(5) $0 \leq F(x) \leq 1$

(6) $F(x)$ is step function دالة متدرجة

(7) $F(x)$ is a non decreasing function or ^{monotonic} increasing function.
 دالة التوزيع التراكمي هي دالة غير متناقصة اي دالة متزايدة

(8) الرسم

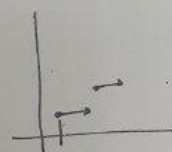


خطوات في الرسم البياني $f(x)$
 $x = a, b, c, d$

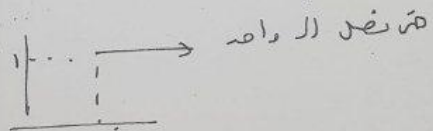
$f(x)$	نبدأ من	x أقل من a
	$p(x=a)$	$a \leq x < b$
	$p(x=a) + p(x=b)$	$b \leq x < c$
	$p(x=a) + p(x=b) + p(x=c)$	$c \leq x < d$
	١ ديكوي جالاجيه داعاً واه	$x \geq d$ x أكبر أو أفظي

خطوات الرسم

$f(x)$



محور افقي يمثل x المتغير المستوي
 محور عمودي يمثل $f(x)$ دالة التوزيع الاحتمالي
 نرسم نقطة النقطة نرسم خط افقي
 من النقطة التي نرسم قسماً الى اليمين بقدر ما
 خطوة خطوة



دالة متزايدة دائماً
 يجب ان نبدأ من الموجب ارن السالب

ex. 1) for example
cx. 7) For the following p.m.f., find the Cumulative dist. fun. and draw it.

$$P(X) = \begin{cases} 1/8 & X=0 \\ 3/8 & X=1 \\ 3/8 & X=2 \\ 1/8 & X=3 \\ 0 & o.w \end{cases}$$

sol $F(x) = P(X \leq x) = \sum_{x \leq x} P(X)$

$$F(x) = \begin{cases} 0 & x < 0 \\ 1/8 & 0 \leq x < 1 \\ 4/8 & 1 \leq x < 2 \\ 7/8 & 2 \leq x < 3 \\ 8/8 = 1 & x \geq 3 \end{cases}$$

$$P(0 \leq x < 1) = P(X=0) = \boxed{1/8}$$

$$P(1 \leq x < 2) = P(X=0) + P(X=1) = \frac{1}{8} + \frac{3}{8} = \boxed{4/8}$$

$$P(2 \leq x < 3) = P(X=0) + P(X=1) + P(X=2) = \frac{1}{8} + \frac{3}{8} + \frac{3}{8} = \boxed{7/8}$$

$$\begin{aligned} P(3 \leq x) &= P(X=0) + P(X=1) + P(X=2) + P(X=3) \\ &= \frac{1}{8} + \frac{3}{8} + \frac{3}{8} + \frac{1}{8} = \underline{8/8} = \boxed{1} \end{aligned}$$

16

drawing the p.m.f.
Figure for p.m.f.

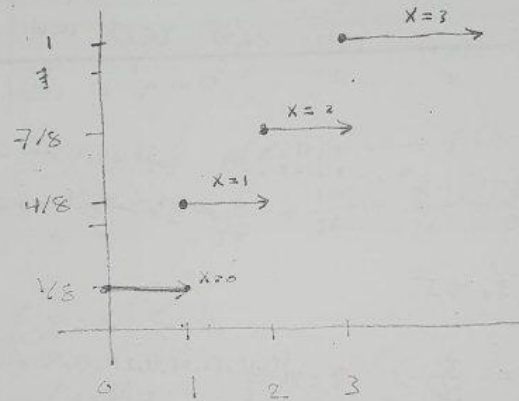
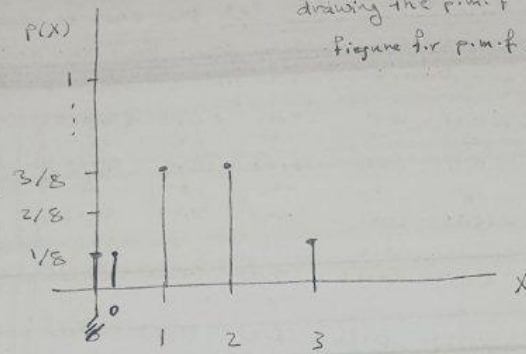


Figure of c.d.f.

EX. 11) For example 5. Find the C.D.F.
 $X =$

$$p(x) = \begin{cases} \frac{1}{4} & x = 0 \\ \frac{2}{4} & x = 1 \\ \frac{1}{4} & x = 2 \\ 0 & \text{o.w.} \end{cases}$$

$p(x)$ is p.m.f.

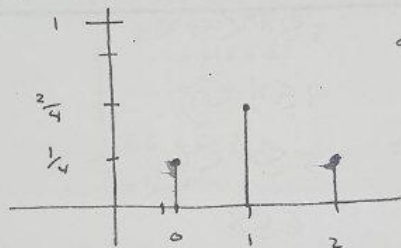
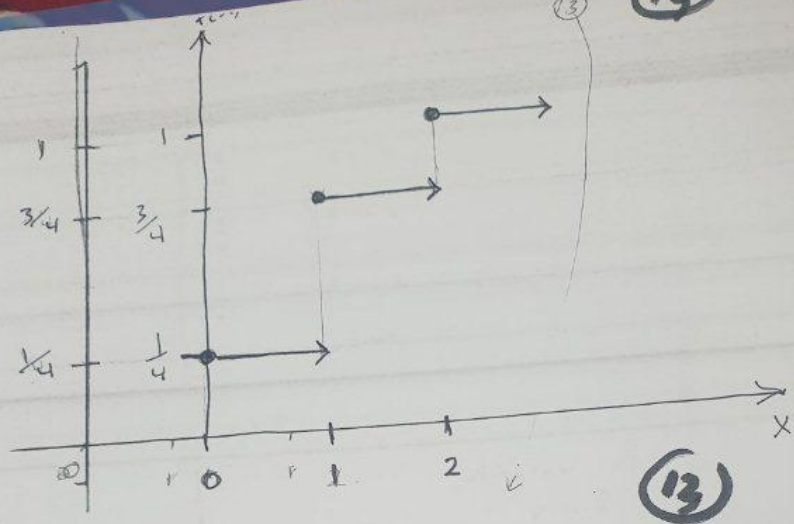


diagram of
p.m.f.

To Find the C.D.F.

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{4} & 0 \leq x < 1 \\ \frac{3}{4} & 1 \leq x < 2 \\ 1 & 2 \leq x \end{cases}$$



EX-13

EX-13 Example 7 Find the cumulative distribution function.

$$p(x) = \begin{cases} \frac{1}{4} & 0 \leq x < 1 \\ \frac{3}{4} & 1 \leq x < 2 \\ 1 & x \geq 2 \end{cases}$$

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{4} & 0 \leq x < 1 \\ \frac{3}{4} & 1 \leq x < 2 \\ 1 & x \geq 2 \end{cases}$$

Graph



Ex. 11 Find the C.D.F. for the $p(x)$ of example 2. (17)

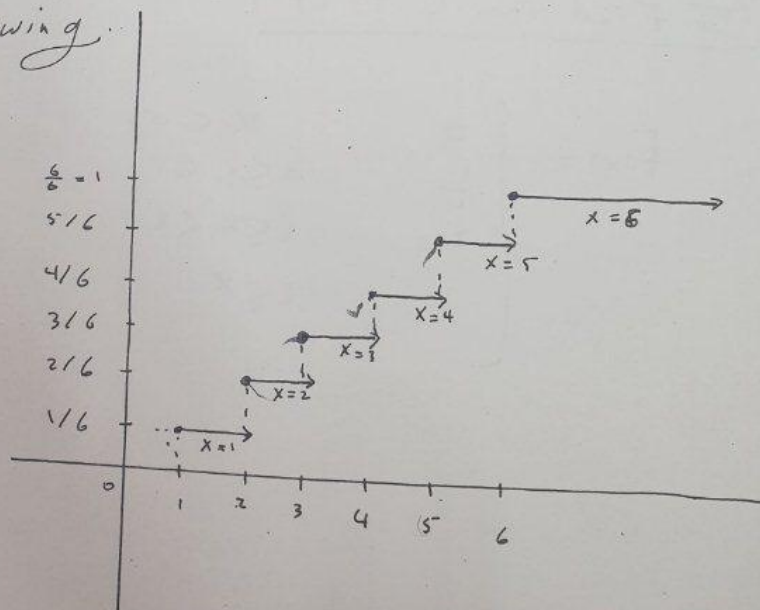
$$x = 1, 2, 3, 4, 5, 6$$

$$p(x) = \begin{cases} \frac{1}{6} & , x = 1, 2, 3, 4, 5, 6 \\ 0 & , \text{o.w.} \end{cases}$$

The C.D.F.

$$F(x) = \begin{cases} 0 & x < 1 \\ \frac{1}{6} & 1 \leq x < 2 \\ \frac{2}{6} & 2 \leq x < 3 \\ \frac{3}{6} & 3 \leq x < 4 \\ \frac{4}{6} & 4 \leq x < 5 \\ \frac{5}{6} & 5 \leq x < 6 \\ \frac{6}{6} = 1 & 6 \leq x \end{cases}$$

The drawing



ex. 9

two dice are thrown once, let X be the sum of the two dice, find C.D.F

from exampl. 3 we have p.m.f.

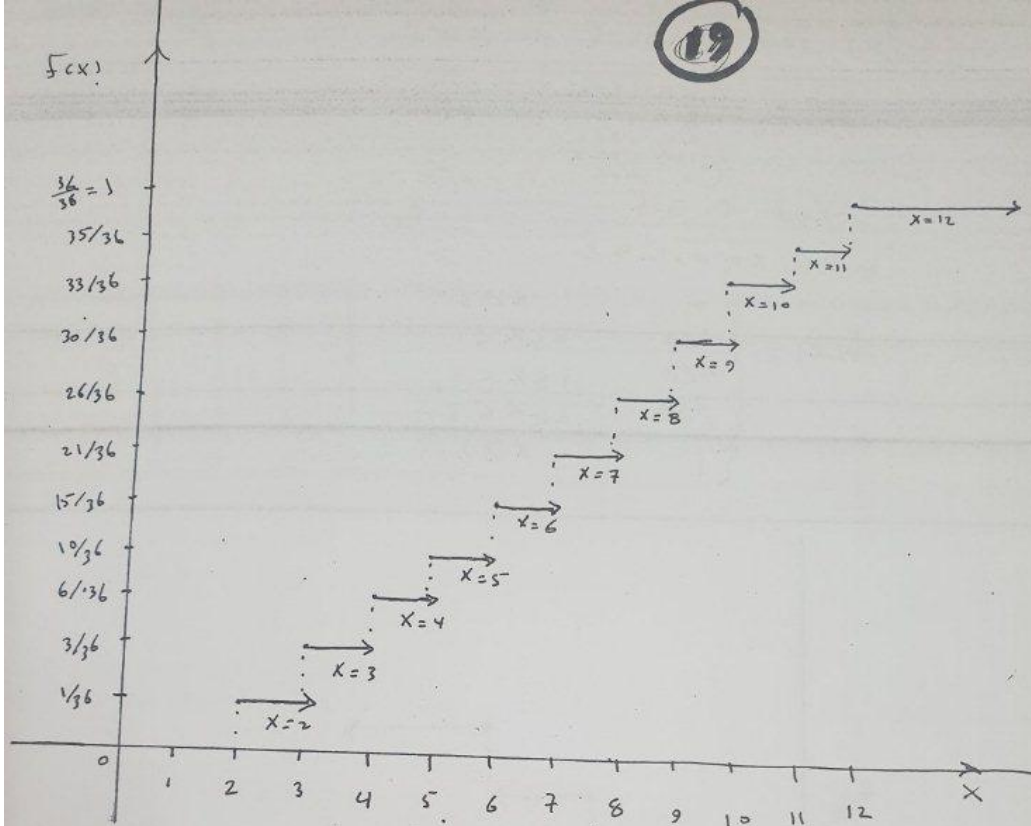
$$p(x) = \begin{cases} \frac{x-1}{36} & x = 2, 3, 4, 5, 6, 7 \\ \frac{13-x}{36} & x = 8, 9, 10, 11, 12 \\ 0 & \text{o.w} \end{cases}$$

and we prove that $p(x)$ is p.m.f.

and drawing it.

Now the c.d.f is.

$$F(x) = \begin{cases} 0 & , x < 2 \\ \frac{1}{36} & , 2 \leq x < 3 \\ \frac{3}{36} & , 3 \leq x < 4 \\ \frac{6}{36} & , 4 \leq x < 5 \\ \frac{10}{36} & , 5 \leq x < 6 \\ \frac{15}{36} & , 6 \leq x < 7 \\ \frac{21}{36} & , 7 \leq x < 8 \\ \frac{26}{36} & , 8 \leq x < 9 \\ \frac{30}{36} & , 9 \leq x < 10 \\ \frac{33}{36} & , 10 \leq x < 11 \\ \frac{35}{36} & , 11 \leq x < 12 \\ \frac{36}{36} = 1 & , 12 \leq x \end{cases}$$



C. D. F.

Ex. 5

Two dice are thrown once, let the r.v. X represent the absolute difference between the two dice. Find the p.m.f and C.D.F. and draw it.

$$S = \left\{ \begin{array}{cccccc} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\ 0 & -1 & -2 & -3 & -4 & -5 \\ (2,1) & (2,2) & (2,3) & (2,4) & (2,5) & (2,6) \\ 1 & 0 & -1 & -2 & -3 & -4 \\ (3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6) \\ 2 & 1 & 0 & -1 & -2 & -3 \\ (4,1) & (4,2) & (4,3) & (4,4) & (4,5) & (4,6) \\ 3 & 2 & 1 & 0 & -1 & -2 \\ (5,1) & (5,2) & (5,3) & (5,4) & (5,5) & (5,6) \\ 4 & 3 & 2 & 1 & 0 & -1 \\ (6,1) & (6,2) & (6,3) & (6,4) & (6,5) & (6,6) \\ 5 & 4 & 3 & 2 & 1 & 0 \end{array} \right\} \quad n(S) = 36$$

المُصنَّف لمُتغيِّر الاختلاف يُعَيَّن: $-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5$ حيث أن X هو المُتغيِّر العشوائي

$$X = 0, 1, 2, 3, 4, 5$$

$$P(X=0) = \frac{6}{36} \rightarrow \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)\} \quad n=6$$

$$P(X=1) = \frac{10}{36} \rightarrow \{(1,2), (2,1), (3,2), (2,3), (4,3), (3,4), (4,5), (5,4), (5,6), (6,5)\} \quad n=10$$

$$P(X=2) = \frac{8}{36} \rightarrow \{(1,3), (3,1), (2,4), (4,2), (5,3), (3,5), (6,4), (4,6)\} \quad n=8$$

$$P(X=3) = \frac{6}{36} \rightarrow \{(1,4), (4,1), (2,5), (5,2), (6,3), (3,6)\} \quad n=6$$

$$P(X=4) = \frac{4}{36} \rightarrow \{(1,5), (5,1), (6,2), (2,6)\} \quad n=4$$

$$P(X=5) = \frac{2}{36} \rightarrow \{(1,6), (6,1)\} \quad n=2$$

In general. p.m.f.

$$p(x) = \begin{cases} 6/36, & x=0 \\ 10/36, & x=1 \\ 8/36, & x=2 \\ 6/36, & x=3 \\ 4/36, & x=4 \\ 2/36, & x=5 \\ 0/36, & \text{o.w} \end{cases}$$

proof the $p(x)$ is p.m.f.

$$\begin{aligned} \textcircled{1} \quad \sum_{x=0}^5 p(X=x) &= p(X=0) + \dots + p(X=5) \\ &= \frac{6}{36} + \frac{10}{36} + \frac{8}{36} + \frac{6}{36} + \frac{4}{36} + \frac{2}{36} = \frac{36}{36} = 1 \end{aligned}$$

$$\textcircled{2} \quad 0 \leq p(X=x) \leq 1$$

$$0 \leq p(X=0), \dots, p(X=5) \leq 1.$$

✓
 $\therefore p(x)$ is p.m.f.

Then the C.D.F

22

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{6}{36} & 0 \leq x < 1 \\ \frac{16}{36} & 1 \leq x < 2 \\ \frac{24}{36} & 2 \leq x < 3 \\ \frac{30}{36} & 3 \leq x < 4 \\ \frac{34}{36} & 4 \leq x < 5 \\ \frac{36}{36} = 1 & 5 \leq x \end{cases}$$

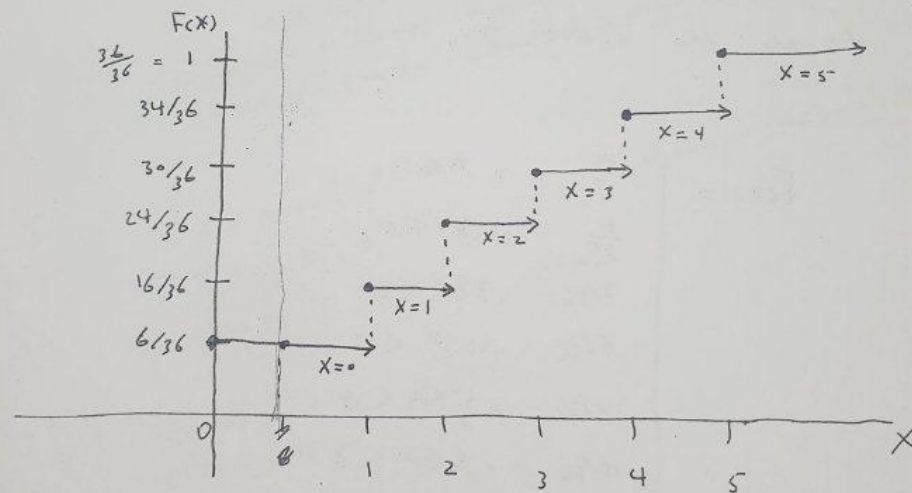
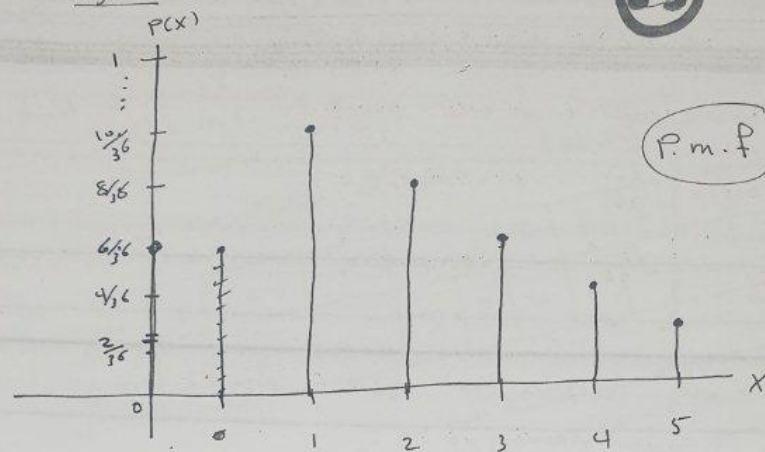
Where

$$F(0 \leq x < 1) = P(X=0) = \frac{6}{36}$$

$$F(1 \leq x < 2) = P(X=0) + P(X=1) = \frac{6}{36} + \frac{10}{36} = \frac{16}{36}$$

⋮

Figure.



Q⁴⁰ we select one integer from the integers 1 through 10. Let the r.v. X be the number of the divisor of each integer.

- Find (i) p.m.f of X
 (ii) c.d.f of X
 (iii) draw it.

نظام التوزيع العشوائي

X عدد مرات تقبل العدد العشوائي

To find the divisor we make the following table.

integer	1	2	3	4	5	6	7	8	9	10
$X = \text{n.o. of divisor}$	1	2	2	3	2	4	2	4	3	4

- الترتيب
 (1) ← يعني العدد يقبل القسمة نفسه فقط مرة واحدة.
 (2) ← يعني العدد يقبل القسمة على واحد وعلى نفسه مثل 2, 3, 5, 7.
 (3) ← يعني العدد يقبل القسمة على واحد وعلى نفسه وعلى عدد آخر مثل 4 و 9 و 6 و 10.
 (4) ← يعني العدد يقبل القسمة على واحد وعلى نفسه وعلى عددين آخرين أي 8 و 12 و 16 و 20.
 وبذلك فإن

$X = \text{n.o. of divisor}$

$X = 1, 2, 3, 4$

$n = 10$

$X = \text{عدد مرات تقبل العدد العشوائي}$

$n = \text{عدد العناصر}$

$P(X=1) = 1/10, X=1$
 $P(X=2) = 4/10, X=2$
 $P(X=3) = 2/10, X=3$
 $P(X=4) = 3/10, X=4$

0

o.w

∴ p.m.f. is

$$p(x) = \begin{cases} 1/10, & x=1 \\ 4/10, & x=2 \\ 2/10, & x=3 \\ 3/10, & x=4 \\ 0, & \text{o.w.} \end{cases}$$

and also $p(x)$ is p.m.f. of the r.v. x , where

$$(i) 0 \leq p(x) \leq 1$$

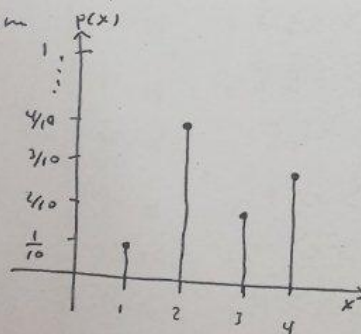
$$(ii) \sum_{x=1}^4 p(x) = p(x=1) + p(x=2) + p(x=3) + p(x=4) \\ = \frac{1}{10} + \frac{4}{10} + \frac{2}{10} + \frac{3}{10} = \frac{10}{10} = 1$$

Hence $p(x)$ is p.m.f. of r.v. x .

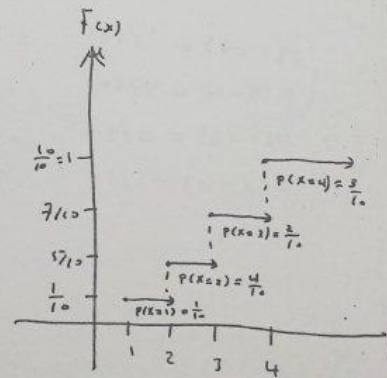
The c.d.f.

$$F(x) = \begin{cases} 0 & x < 1 \\ 1/10 & 1 \leq x < 2 \\ 5/10 & 2 \leq x < 3 \\ 7/10 & 3 \leq x < 4 \\ 10/10 = 1 & 4 \leq x \end{cases}$$

The Diagram



p.m.f. of x .



c.d.f. of x .

EX. 13 For the following function: $P(X) = \begin{cases} \frac{x}{15} & , X=1,2,3,4,5 \\ 0 & \text{o.w.} \end{cases}$

① Find the value of k so that $P(X)$ satisfies the condition of being a probability mass function for a r.v. X ?

② Find the c.d.f. of X ?

$$\textcircled{1} \quad \sum_{\text{all } x} P(X) = 1$$

$$\sum_{x=1}^5 \frac{x}{k} = 1$$

$$\frac{1}{k} \sum_{x=1}^5 x = 1 \quad \rightarrow \text{نحسب المجموع}$$

$$k = \sum_{x=1}^5 x$$

$$k = (1+2+3+4+5)$$

$$\boxed{k = 15}$$

$$\therefore P(X) = \begin{cases} \frac{x}{15} & , X=1,2,3,4,5 \\ 0 & \text{o.w.} \end{cases}$$

$\therefore P(X)$ is p.m.f.

$$\textcircled{2} \quad P(X) = \begin{cases} 1/15 & , X=1 \\ 2/15 & , X=2 \\ 3/15 & , X=3 \\ 4/15 & , X=4 \\ 5/15 & , X=5 \\ 0 & \text{o.w.} \end{cases}$$

$$F(x) = \begin{cases} 0 & x < 1 \\ 1/15 & 1 \leq x < 2 \\ 3/15 & 2 \leq x < 3 \\ 6/15 & 3 \leq x < 4 \\ 10/15 & 4 \leq x < 5 \\ 15/15 & 5 \leq x \end{cases}$$

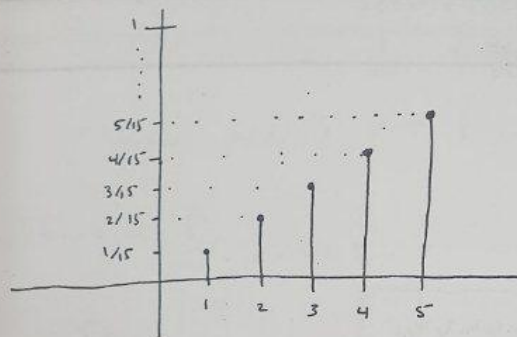


figure of p.m.f.

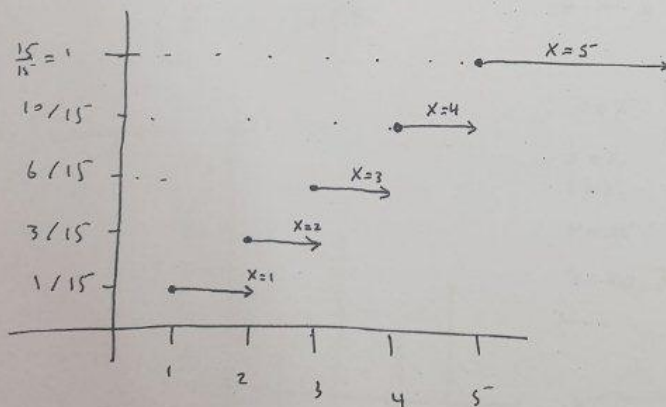


figure of C.D.F.

Q.18

A bowl contains three red chips and five blue chips, two are drawn at random from the bowl, Let X denote the number of red chips in the sample;

(i) Find the p.m.f. of X and draw corresponding graph.

(ii) Find the dist. fun. of X (c.d.f) and graph it.

(i) $X = \text{no. of red chips.}$

$X = 0, 1, 2.$

$$P(X=0) = \frac{C_2^5}{C_2^8} = \frac{10}{28}$$

$$P(X=1) = \frac{C_1^3 C_1^5}{C_2^8} = \frac{15}{28}$$

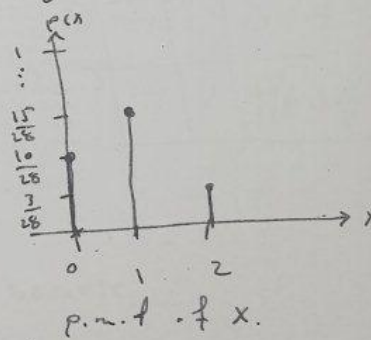
$$P(X=2) = \frac{C_2^3}{C_2^8} = \frac{3}{28}$$

$$P(X) = 0 \quad \dots \dots$$

$$\therefore P(X) = \begin{cases} \frac{10}{28} & , X=0 \\ \frac{15}{28} & , X=1 \\ \frac{3}{28} & , X=2 \\ 0 & \dots \dots \end{cases}$$

لون الرقعة	X عدد الرقعة الحمراء	X	p(X=x _i)
BB زرقة زرقة	0	0	$P(X=0) = \frac{C_1^3 C_2^5}{C_2^8} = \frac{10}{28}$
RR زرقة حمراء	1	1	$P(X=1) = \frac{C_1^3 C_1^5}{C_2^8} = \frac{15}{28}$
RR حمراء حمراء	2	2	$P(X=2) = \frac{C_2^3}{C_2^8} = \frac{3}{28}$

Figure



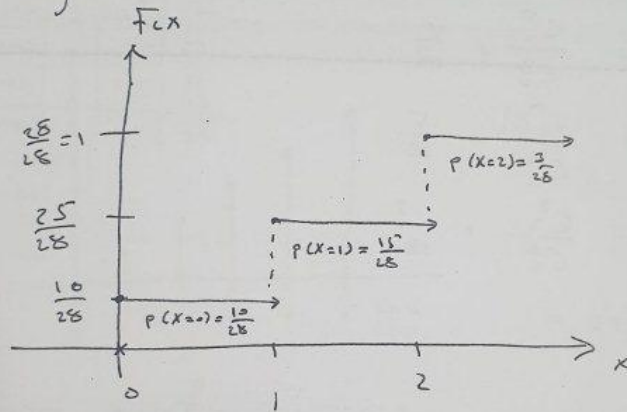
c.d.f. of X or
dist. fun. of X .

29

$X=0, 1, 2$

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{10}{28} & 0 \leq x < 1 \\ \frac{25}{28} & 1 \leq x < 2 \\ \frac{28}{28} = 1 & 2 \leq x \end{cases}$$

The figure.



c.d.f. of X .

Q. Let the random variable X , be the number of white balls of a sample of size 2 drawn without replacement from an urn containing 6 balls of which 4 are white, in which the white balls have been numbered 1 to 4 and the remaining 2 balls 5 and 6. Find the p.m.f., c.d.f.

Sol.

The sample space

$P_2^6 = 30$ possible ways.

$$S = \left\{ \begin{array}{l} (1, 2), (1, 3), \dots, (1, 6) \\ (2, 1), (2, 3), \dots, (2, 6) \\ \vdots \\ (6, 1), (6, 2), (6, 3), \dots, (6, 6) \end{array} \right\}$$

$$n(S) = 30$$

$$X = 0, 1, 2, 3$$

$$P(X=0) = \frac{2}{30} = \frac{1}{15}, X=0$$

$$P(X=1) = \frac{16}{30}, X=1$$

$$P(X=2) = \frac{12}{30}, X=2$$

Then p.m.f. is

$$p(x) = \begin{cases} 2/30, & X=0 \\ 16/30, & X=1 \\ 12/30, & X=2 \end{cases}$$

Also $p(x)$ is p.m.f. of r.v. X because

$$(1) \quad 0 \leq p(x) \leq 1$$

$$(2) \quad \sum_{x=0}^2 p(x) = p(x=0) + p(x=1) + p(x=2) = \frac{2}{30} + \frac{16}{30} + \frac{12}{30} = \frac{30}{30} = 1$$

Hence $p(x)$ is p.m.f.

نوع العينة	X	عدد العينات	احتمال
HH	2	6	$\frac{6}{15} = \frac{2}{5}$
HT	1	8	$\frac{8}{15}$
TH	1	8	$\frac{8}{15}$
TT	0	6	$\frac{6}{15} = \frac{2}{5}$

نوع العينة = X
 ولكن العين المختارة 2 فقط.
 فما سبب كونها 8؟
 أو كونها 6؟
 أو كونها 12؟

$$X = 0, 1, 2$$

نوع العينة	X	P(X=x)
(6,5), (5,6)	0	$P(X=0) = \frac{2}{15}$
(1,2), (2,1), (3,4), (4,3), (5,6), (6,5), (2,3), (3,2), (4,5), (5,4), (6,1), (1,6), (3,6), (6,3)	1	$P(X=1) = \frac{16}{30}$
(1,3), (3,1), (1,4), (4,1), (2,4), (4,2), (3,5), (5,3), (4,6), (6,4), (5,1), (1,5), (6,2), (2,6)	2	$P(X=2) = \frac{12}{30}$

$$n(S) = 30$$

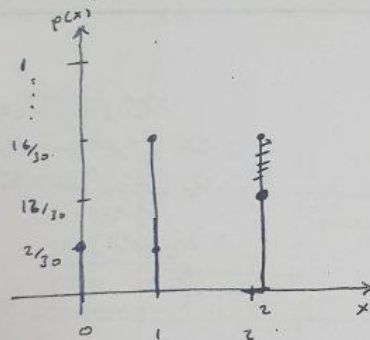
The c.d.f.

$$X = 0, 1, 2.$$

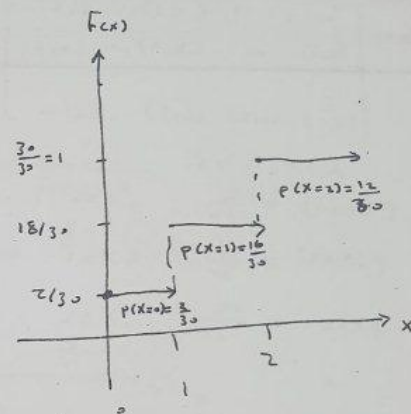
$$F(x) = \begin{cases} 0 & x < 0 \\ 2/30 & 0 \leq x < 1 \\ 18/30 & 1 \leq x < 2 \\ \frac{30}{30} = 1 & 2 \leq x \end{cases}$$

The c.d.f. of X .

The figure.



p.m.f. of X .



c.d.f. of X .

Q/

The r.v. X has the p.m.f

$$P(X) = \begin{cases} \frac{1}{3} & , x=0,1,2 \\ 0 & , \text{o.w} \end{cases}$$

what is the dist. fun. of X ?

The c.d.f. of X ?

we know that.

$$P(X) = \begin{cases} \frac{1}{3} & x=0 \\ \frac{1}{3} & x=1 \\ \frac{1}{3} & x=2 \\ 0 & \text{o.w} \end{cases}$$

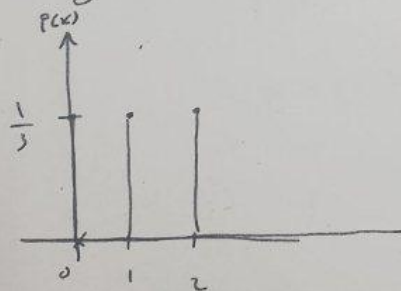
then $x=0,1,2$

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{3} & 0 \leq x < 1 \\ \frac{2}{3} & 1 \leq x < 2 \\ \frac{3}{3} = 1 & 2 \leq x \end{cases}$$

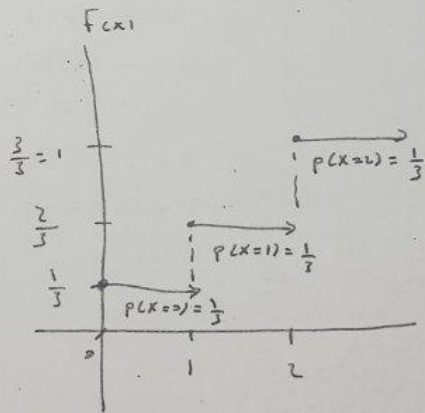
or we can find c.d.f. by p.m.f

$$\begin{cases} \frac{1}{3} \rightarrow \frac{1}{3} & x=0 \\ \frac{2}{3} \rightarrow \frac{2}{3} & x=1 \\ 1 \rightarrow \frac{3}{3} = 1 & x=2 \end{cases}$$

The figure



p.m.f. of X .



c.d.f. of X .

ملف - الفيزياء

- (1) $P(X \text{ exactly } a) = P(X=a)$ ~~or imp.~~
- (2) $P(X \text{ more than } a) = P(X > a) = 1 - P(X \leq a)$
- (3) $P(X \text{ less than } a) = P(X < a)$
- (4) $P(X \text{ at least } a) = P(X \geq a) = 1 - P(X < a)$
- (5) $P(X \text{ at most } a) = P(X \leq a)$
- (6) $P(X \text{ greater than } a) = P(X > a) = 1 - P(X \leq a)$

(7) $P(a < X < b)$

(8) $P(a < X \leq b)$

(9) $P(a \leq X < b)$

(10) $P(a \leq X \leq b)$

~~0/ + C.D.F. of (2x8) 80 = 30/100~~

$$P(a < X \leq b) = F(b) - F(a)$$

$$P(X \leq b) = F(b)$$

$$P(X \leq a) = F(a)$$

$$P(X \leq b) = P(X \leq a) + P(a < X \leq b)$$

34

32 up 17.6

Note

$$P(a < X \leq b) = F(b) - F(a).$$

where

$$P(X \leq b) = P(X \leq a) + P(a < X \leq b)$$

$$F(b) = P(X \leq b)$$

$$F(a) = P(X \leq a).$$

$$F(1) = P(X \leq 1)$$

$$F(2) = P(X \leq 2)$$

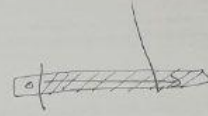
and so on.

The area: compute.

By using p.m.f.

$$(1) \quad P(X > 0) = P$$

$$= \sum_{x=0}^{\infty} P(X)$$



$$(2) \quad P(X > 0) = 1 - P(X \leq 0)$$

$$= 1 - P(0)$$

$$(3) \quad P(a < X < b) =$$

مجموع كل القيم الاحتمالية
المحتوية بين قيم
a و b. وذلك لان قيمة التمام (1) هي 1.



$$(4) \quad P(a \leq X < b) =$$

من ضمنها القيمة
الاحتمالية لـ a



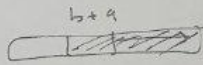
$$(5) \quad P(a < X \leq b)$$



$$(6) \quad P(a \leq X \leq b)$$



$$(7) \quad P(X - a > b) = P(X > b + a)$$



$$(8) \quad P(X \leq b) = P(X \leq a) + P(a < X \leq b)$$

By using c.d.f.

36

$$(1) \quad P(a < X \leq b) = F(b) - F(a).$$

$$(2) \quad P(X \leq b) = F(b)$$

$$P(X \leq a) = F(a).$$

$$P(X \leq 1) = F(1)$$

$$P(X \leq 2) = F(2)$$

and so on.

$$(3) \quad P(X \geq b) = 1 - P(X \leq b) \\ = 1 - F(b)$$

(4)

for example (15) find from (1) to (15)

$$p(x) = \begin{cases} 1/8 & x=0 \\ 3/8 & x=1 \\ 3/8 & x=2 \\ 1/8 & x=3 \\ 0 & \text{o.w.} \end{cases}$$

$$F(x) = \begin{cases} 0 & x < 0 \\ 1/8 & 0 \leq x < 1 \\ 4/8 & 1 \leq x < 2 \\ 7/8 & 2 \leq x < 3 \\ 1 & 3 \leq x \end{cases}$$

$$p(x > 0) = p(1) + p(2) + p(3) \\ = \frac{3}{8} + \frac{3}{8} + \frac{1}{8} \\ = \frac{7}{8}$$

$$p(x > 0) = p(0 \leq x \leq 3) \\ = F(3) - F(0) \\ = 1 - \frac{1}{8} = \frac{7}{8}$$

$$\text{or } p(x > 0) = 1 - p(x \leq 0) \\ = 1 - p(0) \\ = 1 - \frac{1}{8} = \frac{7}{8}$$

$$p(x > 0) = 1 - p(x \leq 0) \\ = 1 - F(0) \\ = 1 - \frac{1}{8} = \frac{7}{8}$$

$$p(x > 2) = p(3) \\ = \left(\frac{1}{8}\right)$$

$$p(x > 2) = p(2 < x \leq 3) \\ = F(3) - F(2) \\ = 1 - \frac{7}{8} = \left(\frac{1}{8}\right)$$

$$\text{or } p(x > 2) = 1 - p(x \leq 2) \\ = 1 - (p(2) + p(1) + p(0)) \\ = 1 - \left(\frac{3}{8} + \frac{3}{8} + \frac{1}{8}\right) \\ = 1 - \frac{7}{8} = \left(\frac{1}{8}\right)$$

$$p(x > 2) = 1 - p(x \leq 2) \\ = 1 - F(2) \\ = 1 - \frac{7}{8} \\ = \left(\frac{1}{8}\right)$$

$$P(X > b) = 1 - P(X \leq b) = 1 - F(b)$$

$$P(X < a) = P(X \leq a) + P(a < X \leq b)$$

$$= F(a) + F(b) - F(a)$$

« Utilidad de $F(b)$ más $F(a)$ »

$$A \quad F(2) - F(0)$$

$$\frac{4}{8} - \frac{1}{8} = \frac{3}{8}$$

« B/A »

$$P(0 < X < 2) = \frac{P(0 < X < 1) + P(1 < X < 2)}{F(1) - F(0) + F(2) - F(1)}$$

$$= F(2) - F(0)$$

« A »

$$P(0 < X < 2) = P(X \leq 0) + P(0 < X \leq 2)$$

$$= F(0) + F(2) - F(0)$$

$$= \frac{4}{8}$$

« A »

« Z »

(A)

(Z)

$$P(0 < X \leq 2) = P(1) + P(2)$$

$$= \frac{3}{8} + \frac{3}{8}$$

$$= \frac{6}{8} \quad \text{«Disjunctive Axiom»}$$

$$P(0 < X \leq 2) = F(2) - F(0)$$

$$= \frac{7}{8} - \frac{1}{8}$$

$$= \frac{6}{8}$$

$$P(0 \leq X < 2) = P(0) + P(1)$$

$$= \frac{1}{8} + \frac{3}{8}$$

$$= \frac{4}{8}$$

(A)

$$P(0 \leq X \leq 2) = F(2) - F(0)$$

$$= \frac{1}{8} - \frac{3}{8}$$

$$= P(X \leq 0) + P(0 < X \leq 2)$$

$$= F(0) + F(2) - F(0)$$

$$= \frac{1}{8} + \frac{7}{8} - \frac{1}{8}$$

$$= \frac{7}{8}$$

(B)

$$P(0 \leq X < 2) = F(1) - F(0)$$

(A)

(A)

«Disjunctive Axiom»

«A»

$\frac{1}{2} = \text{بدرجۀ نصف}$
 (1) $P(\text{exactly } 1) = P(X=1)$

$P(X > 1)$

(2) $P(\text{more than } 1) = P(X > 1) = P(X=2) + P(X=3) + \dots$
 أكبر من واحد

(3) $P(X > 1) = P(X > 1) = 1 - P(X \leq 1)$
 أقل من واحد
 $= 1 - (P(X=0) + P(X=1))$

at least

(4) $P(\text{at least } 1) = P(X \geq 1)$
 أقل من واحد

(5) $P(\text{at most } 1) = P(X \leq 1)$
 أكثر من واحد



(6) $P(X \text{ less than } 1) = P(X < 1)$
 أقل من واحد

44

milan

Dr. Auday

Problem for chapter Four Propability

Problem for discret r.v.

Q1: An urn contains 4 balls numbered 1,2,3,4 respectively Let X be the number that occurs if one ball is drawn at random from the urn. Write the p.m.f. and C.d.F of X and draw it ?
Find : $P(1 < X \leq 4)$, $P(X \geq 2)$?

Q2: Verify the following functions are probpability mass function :

$$(I): f(x) = \begin{cases} \left(\frac{1}{2}\right)^x & , x = 1, 2, 3, \dots \\ 0 & , O.W \end{cases} \quad (II): f(x) = \begin{cases} \left(\frac{1}{4}\right)^x & , x = 1, 2, 3, \dots \\ 0 & , O.W \end{cases}$$

Q3: Determine the constant C so that p(x) satisfies the conditions of being a p.m.f. of X :

$$p(x) = \begin{cases} C (X+1)^2 & , x = 0, 1, 2, 3 \\ 0 & , O.W \end{cases}$$

Q4: Let p(x) be the p.m.f. of arandom variable X Find the distribution function F(x) and sketch it s graph Where :

- (I) $p(x)=1$, $x=3$
- (II) $p(x)=1/2$, $x=1, 2$
- (III) $p(x)=X/15$, $x=1, 2, 3, 4, 5$

$$(IV) p(x) = \begin{cases} \frac{1}{4} \left(\frac{3}{4}\right)^x & , x = 0, 1, 2, \dots \\ 0 & , O.W \end{cases}$$

Q5: The random variable u has the probpability mass function:

- (I) $p(u)=1/2$, $u=-3$
- (II) $p(u)=1/6$, $u=0$
- (III) $p(u)=1/3$, $u=4$

What is the distribution function F(u) and sketch its graph?

Q6: Let x be a r. v. with following distribution : Find the C.d.F. of x and draw the diagram ?

$$p(x) = \begin{cases} 1/4 & x = -2 \\ 1/8 & x = 1 \\ 1/2 & x = 2 \\ 1/8 & x = 4 \\ 0 & o.w. \end{cases}$$

Q7: Bowl contains three red chips and five blue chips , Two chips are drawn at random from the bowl. Let X denote the number of red chips in the sample: Find the p.m.f. and the C.d.F. of x and draw it

Q8 : Two dice are thrown once, Let the r.v. X represent the absolute difference between the Two number shown by the dice : (I) Find the p.m.f. and the C.d.F. of x and draw it ? (II) Find the $p(-1 < x < 3)$, $p(x \text{ more than } 2)$, $p(x \text{ at least } 4)$?

Q9 : The random variable u has the probability mass function :
$$p(x) = \begin{cases} \frac{1}{3} & , X = 1, 2, 3, 4 \\ 0 & , O.W \end{cases}$$

- (I) Show that $p(x)$ is a probability mass function ?
(II) What is the distribution function $F(x)$ and sketch its graph?

Q10 : For the following function :
$$p(x) = \begin{cases} C \cdot x & , X = 1, 2, 3, 4 \\ 0 & , O.W \end{cases}$$

- Find (I) Find the value of the constant (C) so that $p(x)$ satisfies the conditions of being a p.m.f. for a r. v. X ?
(II) C.d.F. of X and draw it?
(III) $E(2X-3)$?
(IV) $\text{Var}(3X+1)$?

Q11: For the following function :
$$p(x) = \begin{cases} a \left(\frac{1}{3} \right)^x & , X = 1, 2, 3, \dots \\ 0 & , O.W \end{cases}$$

- (I) Find the value of the constant a so that $p(x)$ satisfies the conditions of being a p.m.f. for a r. v. X ?

Q12: Let x is a r. v. having the p.m.f.:

x	-2	-1	0	2	3	5
$p(x)$	C	$2C$	$3C$	$2C$	C	$4C$

- (I) Find the value of C ?
(II) Find C.d.F. of x and draw it?
(III) Find : $p(x > -2)$, $p(x < -2)$, $p(x > 0)$, $p(x \text{ more than } 2)$, $p(x \text{ at least } 2)$,
 $P(-1 < X \leq 3)$, $P(-1 < X \leq 3)$, $P(X \geq -2)$, $P(X \leq 5)$?

Q13 : If the C.d.F of x is :
$$F(x) = \begin{cases} 0 & X < 0 \\ 10/28 & 0 \leq X < 1 \\ 25/28 & 1 \leq X < 2 \\ 1 & X \geq 2 \end{cases}$$

- (I) Find the p.m.f. of x ? (II) Find $P(-1 < X < 2)$? (II) Find $P(X-3 \geq -1)$?

Q14: If x is a r. v. having the following p.m.f.:

x	-2	1	3	4
$p(x)$	a^2	$2a^2$	a	a

(I) Show that $a=1/3$?

(II) Find the C.d.F. of x and draw it?

(III) Find: $p(x > 4)$, $p(x < 2)$, $p(x = 2)$, $p(x = -2)$, $p(x = 1)$, $p(x \text{ more than } 2)$, $p(x \text{ at least } 2)$, $P(0 \leq X \leq 2)$, $p(x \text{ at least one})$?

Q15: A die is thrown once. Define a r.v. X is the number shown by the die. (I) Find the p.m.f. and C.d.F. of X and draw it? (II) Find: $p(x > 4)$?

Q16: If the integer from 10 to 20 and we select one at random, and x be the number of divisor of each integer, (I) Find the p.m.f. and the C.d.F. of x and draw it? (II) Find the $p(1 < x < 5)$, $p(x \text{ more than } 2)$, $p(x \text{ at least } 5)$?

Q17: If the integer from 1 to 15 and we select one at random, and x be the number of divisor of each integer, (I) Find the p.m.f. and the C.d.F. of x and draw it? (II) Find the $p(1 < x < 4)$, $p(x \text{ more than } 2)$, $p(x \text{ at least } 3)$?

Q18: If the integer from 1 to 12 and we select one at random, and x be the number of divisor of each integer, (I) Find the p.m.f. and the C.d.F. of x and draw it? (II) Find the $p(1 < x < 4)$, $p(x \text{ more than } 2)$, $p(x \text{ at least } 5)$?

Q19: Let x be a r. v. having the p.m.f.:

$$f(x) = \begin{cases} K \left(\frac{3}{4} \right)^x, & x = 0, 1, 2, 3, \dots \\ 0, & \text{O.W.} \end{cases}$$

(I) Find the value of the constant (K) ? (II) C.d.F. of X ?

Q20: Let Z be a r. v. giving the number of heads minus the number of tails in two tosses of a fair coin. Find the probability function of X ? find the distribution function of X ?

Q21: A die is tossed 100 times. the following table list sex numbers and frequency with Which each number appeared:

no.	1	2	3	4	5	6
frequency	14	17	20	18	15	16

(I) Find the relative frequency of the table?

(II) Find the relative frequency of the events? (i) a 3 appears. (ii) a 5 appears. (iii) an even number appears. (iv) a prime number appears.

(III) Find the mode of the X .

(IV) Find the C.d.F. of x and sketch the graph of $f(x)$ and $F(x)$

Q22 : If x is a r. v. having the p.m.f. : $p(x) = \begin{cases} 3Kx - 2 & , X = 2, 4 \\ 0 & , O.W \end{cases}$

- (I) Find the value of K ?
 (II) Find C.d.F. of x and draw it?
 (III) Find the $p(x \text{ least than } 2)$?

Q23 : If x is a r. v. having the p.m.f. : $p(x) = \begin{cases} Kx - 1 & , X = 3, 4, 5, 6 \\ 0 & , O.W \end{cases}$

- (I) Find the value of K ?
 (II) Find C.d.F. of x and draw it?
 (III) Find the $P(1 \leq X \leq 3)$?

Q24: (A): If x is a r. v. having the p.m.f. : $p(x) = \begin{cases} Kx^2 & , X = 1, 2, 3, 4, 5 \\ 0 & , O.W \end{cases}$

- (I) Find the value of K ?
 (II) Find C.d.F. of x and draw it?
 (III) Find the $P(3 \leq X)$?

Q25: Let : $p(x) = \begin{cases} \frac{(Kx + 1)}{6} & , X = -2, -1, 0, 1 \\ 0 & , O.W \end{cases}$

- (i) Find the value of K Such that $P(X)$ is a p.m.f. ?
 (ii) Find the c.d.f. of X .

Q26 : Find the values of the random variable of each of the following : (I) The absolute difference of the two numbers on the two dice . (II) The number of times that a coin is tossed until ahead appears .

Q27: A pair of fair die is tossed , Let x be a random variable defined as follows : $X(a,b) = \text{Max}(a,b), (a,b) \in S$
 What is the p.m.f. of x ? Derive the C.d.F. of x ?

Q28 : Let x be a r.v. having the C.d.F: $F(x) = \begin{cases} 0 & X < -1 \\ 1/9 & -1 \leq X < 0 \\ 3/9 & 0 \leq X < 1 \\ 1 & 1 \leq X \end{cases}$

- (I) Find the p.m.f. of x ? (II) Find $P(X \geq 0)$, $P(-1 < X < 2)$, $P(-5 \leq X < 1)$, $P(X - 3 \geq -1)$?
 $P(1 < X - 1 > 2)$