وزارة التعليم العالي والبحث العلمي
الجامعة المستنصرية
كلية الإدارة و الإقتصاد
قسم الإحصاء

الاستدلال الاحصائي (التقديرات)

- Definition of estimation.
- Graphical estimation.
- Method of point estimation.
- Unbiasedness.
- Mean squared error.
- Consistency.
- Sufficient statistics.
- Rao–black well theorem.
- Crammer Rao inequality.
- Introduction and definition.
- Confidence interval for mean.
- Confidence interval for differ.
- Confidence interval for variance.
- Confidence interval for ratio.
- Applications.

استاذ المادة (1) ، (2)
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2019-2020 (الكورس الاول)

1 - البروفايل الخاص بالأستاذ:

https://uomustansiriyah.edu.iq/e-learn/profile.php?id=3290

2- مدخل في الاستدلال الاحصائي ، د. عبد المجيد حمزة الناصر

Example (5): Let $X_1, X_2, \dots, X_n$ be a r.s from a Bernoulli distribution.

Estimate the parameter P by using:

1) method of moment.
2) MLE Method.

Solution: $X \sim Ber(1, p) \rightarrow E(X) = p$ **and** $var(X) = E(X^2) - (E(X))^2 = P(1-P)$

$$f(x; p) = P^x (1-P)^{1-x} \quad , x = 0,1 \quad , \text{Zero otherwise}$$

First:

$$1)\ \mu_r = E(X^r) \rightarrow \qquad \mu_1 = E(X) = P$$

$$2)\ \hat{\mu}_r = \frac{\sum X_i^{\,r}}{n} \rightarrow \qquad \hat{\mu}_1 = \frac{\sum X_i}{n} = \bar{X}$$

$$3)\ \mu_1 = \hat{\mu}_1 \rightarrow \boxed{\hat{P} = \bar{X}}$$

$\therefore$ $\bar{X}$ **is moment estimator for** P

Second:

$$L(P; X_1, X_2, \dots, X_n) = f(X_1, X_2, \dots, X_n; P)$$

$$= f(X_1; P) * f(X_2; P) * \dots * f(X_n; P) = \prod_{i=1}^{n} f(X_i; P)$$

$$L = [P^{x_1}(1-P)^{1-x_1}] * \dots * [P^{x_n}(1-P)^{1-x_n}] = P^{\sum x_i}(1-P)^{n-\sum x_i}$$

$$\text{Ln } L = \sum x_i \ln(P) + \left(n - \sum x_i\right) \ln(1-P)$$

$$\frac{\partial \text{Ln } L}{\partial P} = \left(\sum x_i\right)\frac{1}{P} + \left(n - \sum x_i\right)\frac{1}{(1-P)}(-1) = \frac{\sum x_i}{P} - \frac{(n - \sum x_i)}{1-P}$$

$$\frac{\partial \text{Ln } L}{\partial P} = 0$$

$$\frac{\sum x_i}{\hat{P}} - \frac{(n - \sum x_i)}{1-\hat{P}} = 0 \rightarrow \frac{(1-\hat{P})\sum x_i - \hat{P}(n - \sum x_i)}{\hat{P}(1-\hat{P})} = 0$$

$$\therefore \sum x_i - \hat{P}\sum x_i - n\hat{P} + \hat{P}\sum x_i = 0 \rightarrow \sum x_i = n\hat{P} \rightarrow \boxed{\hat{P} = \bar{X}}$$

$\therefore$ $\bar{X}$ **is the MLE for** P

Example (6): Let $X_1, X_2, \dots, X_n$ be a r.s from the negative exponential distribution.

Estimate the parameter θ by using:

1) method of moment.
2) MLE Method.

Solution: $X \sim \exp(\theta) \rightarrow E(X) = \frac{1}{\theta}$ **and** $var(X) = E(X^2) - (E(X))^2 = \frac{1}{\theta^2}$

$$f(x,\theta) = \theta\, e^{-\theta X} \quad , I(0,\infty)^{(X)} \quad , \text{Zero otherwise}$$

First:

1) $\mu_r = E(X^r) \rightarrow \qquad \mu_1 = E(X) = \frac{1}{\theta}$

2) $\hat{\mu}_r = \frac{\sum X_i{}^r}{n} \rightarrow \qquad \hat{\mu}_1 = \frac{\sum X_i}{n} = \bar{X}$

3) $\mu_1 = \hat{\mu}_1 \rightarrow \frac{1}{\hat{\theta}} = \bar{X} \rightarrow \boxed{\hat{\theta} = \frac{1}{\bar{X}}}$

$\therefore \frac{1}{\bar{X}}$ **is moment estimator for θ**

Second:

$$L(\theta; X_1, X_2, \dots, X_n) = f(X_1, X_2, \dots, X_n; \theta)$$

$$= f(X_1;\theta) * f(X_2;\theta) * \dots * f(X_n;\theta) = \prod_{i=1}^{n} f(X_i;\theta)$$

$$L = [\theta\, e^{-\theta X_1}] * \dots * [\theta\, e^{-\theta X_n}] = \theta^n e^{-\theta \sum x_i}$$

$$\text{Ln } L = n\text{Ln}(\theta) - \theta \sum x_i$$

$$\frac{\partial \text{Ln } L}{\partial \theta} = (n)\frac{1}{\theta} - \sum x_i$$

$$\frac{\partial \text{Ln } L}{\partial \theta} = 0$$

$$\frac{n}{\hat{\theta}} - \sum x_i = 0 \rightarrow \frac{n}{\hat{\theta}} - \frac{\hat{\theta} \sum x_i}{\hat{\theta}} = 0 \rightarrow n = \hat{\theta} \sum x_i \rightarrow \hat{\theta} = \frac{n}{\sum x_i} = \frac{\mathbf{1}}{\bar{\bar{X}}}$$

$$\therefore \boxed{\hat{\theta} = \frac{1}{\bar{\bar{X}}}}$$

$\therefore \frac{1}{\bar{X}}$ **is the MLE for θ**

Example (7): Let $X_1, X_2, \dots, X_n$ be a r.s from Normal distribution.

Estimate the two parameters μ and σ^2 by using:

1) method of moment.
2) MLE Method.

Solution: $X \sim N(\mu, \sigma^2) \rightarrow E(X) = \mu$ and $var(X) = \mathbf{E(X^2)} - (\mathbf{E(X)})^2 = \sigma^2$

$$f(x, \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\frac{(X-\mu)^2}{\sigma^2}} \quad , -\infty \le x \le \infty \quad , \text{Zero otherwise}$$

First: (مكرر)

1) $\mu_r = E(X^r) \rightarrow \qquad \mu_1 = E(X) = \mu$

2) $\hat{\mu}_r = \frac{\sum X_i^r}{n} \rightarrow \qquad \hat{\mu}_1 = \frac{\sum X_i}{n} = \bar{X}$

3) $\mu_1 = \hat{\mu}_1 \rightarrow \boxed{\hat{\mu} = \bar{X}}$

4) $\mu_r = E(X^r) \rightarrow \qquad \mu_2 = E(X^2) = var(X) + (E(X))^2 = \sigma^2 + \mu^2$

5) $\hat{\mu}_r = \frac{\sum X_i^r}{n} \rightarrow \qquad \hat{\mu}_2 = \frac{\sum X_i^2}{n}$

6) $\mu_2 = \hat{\mu}_2 \rightarrow \hat{\sigma}^2 + \hat{\mu}^2 = \frac{\sum X_i^2}{n} \rightarrow \hat{\sigma}^2 = \frac{\sum X_i^2}{n} - \bar{X}^2$

$$\boxed{\hat{\sigma}^2 = S^2 = \frac{\sum (X_i - \bar{X})^2}{n}} \quad , \text{where} \sum (X_i - \bar{X})^2 = \sum X_i^2 - n\bar{X}^2$$

$\therefore$ **$\bar{X}$ and S^2 are moment estimators for** μ and σ^2

Second:

$$L(\mu, \sigma^2; X_1, X_2, \dots, X_n) = f(X_1, X_2, \dots, X_n; \mu, \sigma^2)$$

$$= f(X_1; \mu, \sigma^2) * \dots * f(X_n; \mu, \sigma^2) = \prod_{i=1}^{n} f(X_i; \mu, \sigma^2)$$

$$L = \left[\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\frac{(X_1-\mu)^2}{\sigma^2}}\right] * \dots * \left[\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\frac{(X_n-\mu)^2}{\sigma^2}}\right] = \left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^n e^{-\frac{1}{2}\frac{\sum (X-\mu)^2}{\sigma^2}}$$

$$L = (2\pi)^{\frac{-n}{2}} * (\sigma^2)^{\frac{-n}{2}} * e^{-\frac{1}{2}\frac{\sum(X_i-\mu)^2}{\sigma^2}}$$

$$\text{Ln } L = \frac{-n}{2}\text{Ln}(2\pi) - \frac{n}{2}\text{Ln}(\sigma^2) - \frac{1}{2}\frac{\sum(X_i-\mu)^2}{\sigma^2}$$

$$\frac{\partial \text{Ln } L}{\partial \mu} = 0 - 0 - \frac{1}{2}\frac{\sum(X_i-\mu)}{\sigma^2}(-2) = \frac{\sum(X_i-\mu)}{\sigma^2}$$

$$\frac{\partial \text{Ln } L}{\partial \mu} = 0$$

$$\frac{\sum(X_i-\hat{\mu})}{\hat{\sigma}^2} = 0 \rightarrow \sum X_i - n\hat{\mu} = 0$$

$$\therefore \boxed{\hat{\mu} = \bar{\mathbf{X}}}$$

$\therefore \bar{\mathbf{X}}$ **is the MLE for μ**

$$\text{Ln } L = \frac{-n}{2}\text{Ln}(2\pi) - \frac{n}{2}\text{Ln}(\sigma^2) - \frac{1}{2}\frac{\sum(X_i-\mu)^2}{\sigma^2}$$

$$\frac{\partial \text{Ln } L}{\partial \sigma^2} = 0 - \frac{n}{2}\frac{1}{\sigma^2} - \frac{\sum(X_i-\mu)^2}{2}\left(\frac{-1}{\sigma^4}\right) = \frac{-n}{2\sigma^2} + \frac{\sum(X_i-\mu)^2}{2\sigma^4}$$

$$\frac{\partial \text{Ln } L}{\partial \sigma^2} = \frac{-n}{2\hat{\sigma}^2} + \frac{\sum(X_i-\hat{\mu})^2}{2\hat{\sigma}^4} = 0 \rightarrow \frac{-n\hat{\sigma}^2 + \sum(X_i-\bar{\mathbf{X}})^2}{2\hat{\sigma}^4} = 0$$

$$\therefore n\hat{\sigma}^2 = \sum(X_i-\bar{X})^2 \rightarrow \hat{\sigma}^2 = \frac{\sum(X_i-\bar{X})^2}{n} = S^2$$

$\therefore \hat{\sigma}^2 = S^2 = \frac{\sum(X_i-\bar{X})^2}{n}$ **is the MLE for** σ^2